BLP

Demand side:

$$U(\zeta_i, p_j, x_j, \xi_j; \theta),$$

$$U(\zeta_i, p_j, x_j, \xi_j; \theta) \geq U(\zeta_i, p_r, x_r, \xi_r; \theta), \quad \text{for} \quad r = 0, 1, \ldots, J,$$

$$A_j = \{\zeta : U(\zeta, p_j, x_j, \xi_j; \theta) \geq U(\zeta, p_r, x_r, \xi_r; \theta), \text{for } r = 0, 1, \ldots, J\}.$$

$$s_j(p, x, \xi; \theta) = \int_{\zeta \in A_j} P_0(d\zeta).$$

M – market capacity

Ms – demand function

$$U(\zeta_i, p_j, x_j, \xi_j; \theta) \equiv x_j \beta - \alpha p_j + \xi_j + \epsilon_{i,j} \equiv \delta_j + \epsilon_{ij},$$

$$s_j = \int_\epsilon \prod_{q \neq j} P(\delta_j - \delta_q + \epsilon) P(d\epsilon).$$

Not as good as weather due to unexpected substitutional patterns. Random utility is better:

$$U(\zeta_i, p_j, x_j, \xi_j; \theta) = x_j \bar{\beta} - \alpha p_j + \xi_j + \sum_k \sigma_k x_{jk} \nu_{ik} + \epsilon_{ij},$$

$$\delta_j = x_j \bar{\beta} - \alpha p_j + \xi_j$$

and a deviation from that mean

$$\mu_{ij} = \sum_k \sigma_k x_{jk} \nu_{ik} + \varepsilon_{ij},$$

Incorporate observable income distribution (CPS) in model:

$$U(\zeta_i, p_j, x_j, \xi_j; \theta) = (y_i - p_j)^{\alpha} G(x_j, \xi_j, \nu_i) e^{\epsilon(i,j)},$$

$$u_{ij} = \alpha \log(y_i - p_j) + x_j \bar{\beta} + \xi_j + \sum_k \sigma_k x_{jk} \nu_{ik} + \epsilon_{ij},$$

$$u_{i0} = \alpha \log(y_i) + \xi_0 + \sigma_0 \nu_{i0} + \epsilon_{i0}.$$

Supply side:

$$\ln(mc_j) = w_j \gamma + \omega_j,$$

or with return to scale $\ln(mc_j) = w_j \gamma + \mu \ln(q_j) + \omega_j$

$$\Pi_f = \sum_{j \in \mathcal{F}_f} (p_j - mc_j) M s_j(p, x, \xi; \theta),$$

FOC:

$$s_j(p, x, \xi; \theta) + \sum_{r \in \mathcal{F}_f} (p_r - mc_r) \frac{\partial s_r(p, x, \xi; \theta)}{\partial p_j} = 0.$$

$$\Delta_{jr} = \begin{cases} \dfrac{-\partial s_r}{\partial p_j}, & \text{if } r \text{ and } j \text{ are produced by the same firm;} \\ 0, & \text{otherwise.} \end{cases}$$

$$s(p, x, \xi : \theta) - \Delta(p, x, \xi; \theta)[p - mc] = 0.$$

$$p = mc + \Delta(p, x, \xi; \theta)^{-1} s(p, x, \xi; \theta).$$

$$b(p, x, \xi; \theta) \equiv \Delta(p, x, \xi; \theta)^{-1} s(p, x, \xi; \theta).$$

$$\ln(p - b(p, x, \xi; \theta)) = w\gamma + \omega.$$

BLP instruments:

For example, if one of our characteristics is the size of a car, then the instrument vector for product *j* includes the size of car *j*, the sum of size across own-firm products, and the sum of size across rival firm products.

Estimation:

There are four steps to each evaluation of $G_J(\theta, s^n, P_{ns})$ in both models. For each θ:

(i) estimate (via simulation) the market shares implied by the model;

(ii) solve for the vector of demand unobservables [i.e. $\xi(\theta, s^n, P_{ns})$] implied by the simulated and observed market shares;

(iii) calculate the cost side unobservable, $\omega(\theta, s^n, P_{ns})$, from the difference between price and the markups computed from the shares; and finally

(iv) calculate the optimal instruments and interact them with the computed cost and demand side unobservables (as in (5.3)) to produce $G_J(\theta, s^n, P_{ns})$.

$$E[\xi_j|z] = E[\omega_j|z] = 0.$$

$$E\left[(\xi_j, \omega_j)'(\xi_j, w_j)|z\right] = \Omega(z_j)$$

$$T(z)'T(z) = \Omega(z)^{-1}$$

$$G^J(\theta) = E\left[H_j(z)T(z_j)\begin{pmatrix}\xi_j(\theta, s^0, P_0)\\ w_j(\theta, s^0, P_0)\end{pmatrix}\right]$$

Minimize G'G for theta

Data:

Country-level data, 20 years, yearly dynamics, 2217 models/years, 997 distinct models

Income distribution from CPS

M as number of households

Limitations:

1. No variation of consumer characteristics (Done in Nevo)
2. More detailed information on cost function (R&D etc.)
3. Correlation between observed and unobserved characteristics (strategic behavior of firms on differentiation; dynamic industry equilibrium)
4. Dynamic consumer behavior; zero alternative includes used cars

Nevo

Data:

City (45) x quarter (20) x brands (24) = 21600

IRI data on sales (shares) and prices, also on advertising.

Market shares are defined by converting volume sales into number of servings sold and dividing by the total potential number of servings in a city in a quarter. This potential was assumed to be one serving per capita per day. The market share of the outside good was defined as the difference between one and the sum of inside goods market shares

Demographics from CPS

Demand side:

$$u_{ijt} = x_{jt}\beta_i^* + \alpha_i^* p_{jt} + \xi_{jt} + \epsilon_{ijt} \equiv V_{ijt} + \epsilon_{ijt},$$

$$\xi_{jt} = \xi_j + \xi_t + \Delta\xi_{jt}.$$

$$\begin{pmatrix} \alpha_i^* \\ \beta_i^* \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + \Pi D_i + \Sigma v_i, \qquad v_i \sim N(0, I_{K+1}),$$

$$u_{i0t} = \xi_0 + \pi_0 D_i + \sigma_0 v_{i0} + \epsilon_{i0t}$$

$$A_{jt}(x_{\cdot t}, p_{\cdot t}, \xi_{\cdot t}; \theta) = \{(D_i, v_i, \epsilon_{\cdot t}) | u_{ijt} \geq u_{i\ell t} \ \forall \ \ell = 0, 1, \ldots, J\}$$

$$s_{jt}(x_{\cdot t}, p_{\cdot t}, \xi_{\cdot t}; \theta) = \int_{A_{jt}} dP^*(D, v, \epsilon) = \int_{A_{jt}} dP_\epsilon^*(\epsilon)\, dP_v^*(v)\, dP_D^*(D),$$

Instruments for $\Delta\xi_{jt}$:

prices of the same product in different time or market (valid because different market, relevant because of the same component in MC)

Exactly: regional quarterly average prices (excluding the city being instrumented) in all twenty quarters

Supply side:

$$\Pi_f = \sum_{j \in \mathcal{F}_f} (p_j - mc_j) M s_j(p) - C_f,$$

$$s_j(p) + \sum_{r \in \mathcal{F}_f} (p_r - mc_r) \frac{\partial s_r(p)}{\partial p_j} = 0.$$

$$\Omega_{jr}^{pre}(p) = \begin{cases} -\partial s_j(p)/\partial p_r, & \text{if } \exists f\colon \{r, j\} \subset \mathcal{F}_f; \\ 0, & \text{otherwise.} \end{cases}$$

$$s(p) - \Omega^{pre}(p)(p - mc) = 0.$$

$$p - mc = \Omega^{pre}(p)^{-1} s(p) \Rightarrow mc = p - \Omega^{pre}(p)^{-1} s(p).$$

Limitations:

1. Cost savings after merging
2. Nonrandom advertising and brand introduction

Advantages:

1. Computer code in MATLAB. May replicate the results.

BH

Demand side:

Market is defined by $\chi_t = (x_t, p_t, \xi_t)$

v_{ijt} is conditional indirect utility of consumer i

$$F_v(v_{i1t}, \ldots, v_{iJt} | \chi_t).$$

$$s_{jt} = \sigma_j(\chi_t) = \Pr\left(\arg\max_{k \in \mathcal{J}} v_{ikt} = j \middle| \chi_t\right), \quad j = 0, \ldots, J.$$

So far, the only restriction placed on the random utility model is the restriction to a scalar product/market unobservable ξ_{jt} for each t and $j = 1, \ldots, J$. We now add an important index restriction. Partition x_{jt} as $(x_{jt}^{(1)}, x_{jt}^{(2)})$, with $x_{jt}^{(1)} \in \mathbb{R}$. Let $x_t^{(1)} = (x_{1t}^{(1)}, \ldots, x_{Jt}^{(1)})$ and $x_t^{(2)} = (x_{1t}^{(2)}, \ldots, x_{Jt}^{(2)})$. Define the linear indices

$$\delta_{jt} = x_{jt}^{(1)} \beta_j + \xi_{jt}, \quad j = 1, \ldots, J \tag{2}$$

and let $\delta_t = (\delta_{1t}, \ldots, \delta_{Jt})$.

ASSUMPTION 1: $F_v(\cdot|\chi_t) = F_v(\cdot|\delta_t, x_t^{(2)}, p_t)$.

DEFINITION 1: Let λ_t denote either $-p_t$ or δ_t. Goods $(0, 1, \ldots, J)$ are *connected substitutes in* λ_t if both

(i) $\sigma_k(\delta_t, p_t)$ is nonincreasing in λ_{jt} for all $j > 0$, $k \neq j$, $(\delta_t, p_t) \in \mathbb{R}^{2J}$; and

(ii) at each $(\delta_t, p_t) \in \operatorname{supp}(\delta_t, p_t)$, for any nonempty $\mathcal{K} \subseteq \{1, \ldots, J\}$ there exist $k \in \mathcal{K}$ and $\ell \notin \mathcal{K}$ such that $\sigma_\ell(\delta_t, p_t)$ is strictly decreasing in λ_{kt}.

ASSUMPTION 2: *Goods* $(0, 1, \ldots, J)$ *are connected substitutes in* $-p_t$ *and in* δ_t.

ASSUMPTION 3: *For all* $j = 1, \ldots, J$, $E[\xi_{jt}|z_t, x_t] = 0$ *almost surely.*

ASSUMPTION 4: *For all functions* $B(s_t, p_t)$ *with finite expectation, if* $E[B(s_t, p_t)|z_t, x_t] = 0$ *almost surely, then* $B(s_t, p_t) = 0$ *almost surely.*

THEOREM 1: *Under Assumptions* 1–4, *for all* $j = 1, \ldots, J$, (i) ξ_{jt} *is identified with probability* 1 *for all* t, *and* (ii) *the function* $\sigma_j(\chi_t)$ *is identified on* $\mathcal{X}$.

Supply side:

ASSUMPTION 6: (i) $\sigma_j(\chi_t)$ *is continuously differentiable with respect to* p_k $\forall j, k \in \mathcal{J}$; (ii) *at each* $\chi_t \in \mathcal{X}$, *for any nonempty* $\mathcal{K} \subseteq \{1, \ldots, J\}$, *there exist* $k \in \mathcal{K}$ *and* $\ell \notin \mathcal{K}$ *such that* $\partial\sigma_\ell(\chi_t)/\partial p_{kt} > 0$.

Denote $D(\chi_t)$ as JxJ matrix of partial derivatives $[\frac{\partial\sigma_k(\chi_t)}{\partial p_l}]_{k,l}$

ASSUMPTION 7a: *For each* $j = 1, \ldots, J$, *there exists a known function* ψ_j *such that, for all* (M_t, χ_t, s_t) *in their support,*

$$mc_{jt} = \psi_j\big(s_t, M_t, D(\chi_t), p_t\big). \tag{9}$$

THEOREM 2: *Suppose Assumptions* 1–4, 6, *and* 7a *hold. Then* mc_{jt} *is identified for all* t, $j = 1, \ldots, J$.

But ψ_j is known (as was in Nevo)!

Consider

$$mc_{jt} = \tilde{c}_j(Q_{jt}, \mathrm{w}_{jt}, \omega_{jt}),$$ or $$mc_{jt} = \overline{c}_j(Q_{jt}, \mathrm{w}_{jt}) + \omega_{jt},$$

Let y_{jt} abe excluded instruments for Q_{jt}, formally saying:

ASSUMPTION 8: $E[\omega_{jt}|\mathrm{w}_{jt}, y_{jt}] = 0$ *almost surely for all* $j = 1, \ldots, J$.

ASSUMPTION 9: *For all* $j = 1, \ldots, J$ *and all functions* $B(Q_{jt}, \mathrm{w}_{jt})$ *with finite expectation, if* $E[B(Q_{jt}, \mathrm{w}_{jt})|\mathrm{w}_{jt}, y_{jt}] = 0$ *almost surely, then* $B(Q_{jt}, \mathrm{w}_{jt}) = 0$ *almost surely.*

THEOREM 3: *Suppose marginal costs take the form in* (11) *and that Assumptions* 1–4, 6, 7a, 8, *and* 9 *hold. Then for all* $j = 1, \ldots, J$, (i) *the marginal cost functions* $\overline{c}_j(Q_{jt}, \mathrm{w}_{jt})$ *are identified, and* (ii) ω_{jt} *is identified with probability* 1 *for all* t.

Equilibrium:

Assumption 7b: *For each* $j = 1, \ldots, J$, *there exists a (possibly unknown) function* ψ_j *such that, for all* (M_t, χ_t, s_t) *in their support,*

$$mc_{jt} = \psi_j\big(s_t, M_t, D(\chi_t), p_t\big). \tag{12}$$

on the marginal cost function. Partition the cost shifters w_{jt} as $(\mathrm{w}_{jt}^{(1)}, \mathrm{w}_{jt}^{(2)})$, with $\mathrm{w}_{jt}^{(1)} \in \mathbb{R}$, and define the "cost index"

$$\kappa_{jt} = \mathrm{w}_{jt}^{(1)} \gamma_j + \omega_{jt}.$$

Assumption 10: *For all* $j = 1, \ldots, J$, $mc_{jt} = c_j(Q_{jt}, \kappa_{jt}, \mathrm{w}_{jt}^{(2)})$, *where* c_j *is strictly increasing in* κ_{jt}.

$$x_{jt} + \xi_{jt} = \sigma_j^{-1}(s_t, p_t), \quad j = 1, \ldots, J,$$

$$\mathrm{w}_{jt} + \omega_{jt} = \pi_j^{-1}(s_t, p_t), \quad j = 1, \ldots, J.$$

Assumption 13: *There is a unique vector of equilibrium prices associated with any* (δ_t, κ_t).

$$p_t = \pi(\delta_t, \kappa_t) \tag{17}$$

for some unknown function $\pi : \mathbb{R}^{2J} \to \mathbb{R}^J$.

Assumption 14: (ξ_t, ω_t) *have a positive joint density* $f_{\xi,\omega}$ *on* $\mathbb{R}^{2J}$.

Assumption 15: *The function* $\pi(\delta_t, \kappa_t)$ *is differentiable, and the functions* $\sigma^{-1}(s_t, p_t)$ *and* $\pi^{-1}(s_t, p_t)$ *are continuously differentiable.*

Assumption 16: $(x_t, \mathrm{w}_t) \perp\!\!\!\perp (\xi_t, \omega_t)$.

Assumption 17: $\mathrm{supp}(x_t, \mathrm{w}_t) = \mathbb{R}^{2J}$.

Theorem 5: *Suppose Assumptions* 1, 2, 6, 7b, 10, *and* 13–17 *hold. Then, for all* $j = 1, \ldots, J$, (i) ξ_{jt} *is identified for all* t, *and* (ii) *the function* $\sigma_j(\delta_t, p_t)$ *is identified on* $\mathcal{X}$.

Theorem 6: *Suppose the hypotheses of Theorem* 5 *hold. Then* (i) ω_{jt} *is identified for all* t *and* $j = 1, \ldots, J$; (ii) $\pi(\delta_t, \kappa_t)$ *is identified.*