Multiple Discrete-Continuous (MDC) Models of Consumer Demand

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Outline

1. Examples of multiple discreteness (MD)
2. Consumer Theory
3. Discrete choice theory
4. Approaches to MD
5. Bhat's Paper (2008)
    - Model
    - Data
    - Estimation

General Choice Structure

Consumer choice

<u>Discrete Choice</u>

which alternatives
to choose from
available set

Discrete Choice Theory (DCT)

<u>Continuous Choice</u>

which quantities of
chosen alternatives to
select

Neoclassic Consumer Theory (NCT)

Examples of Choice Situations

Consumer choice

Discrete Choice		**Continuous Choice**
Which brands to select	→	How many units of brands to buy
Which different types of activities to invest time in	→	How much time to allocate in each of chosen activities
Which assets to invest in	→	How much to invest in each of chosen activities

Consumer Theory

Assumption: x is continuous

$$\begin{cases} U(p,x) \to max, \\ px = I, \\ x \geq 0 \end{cases}$$

$U(p,x)$ - direct utility function
$p = (p_1, \dots, p_N)$ – unit prices for goods $\mathrm{i} = \overline{1, N}$
$x = (x_1, \dots,\ x_N)^T$ – volumes of consumption
I – budget for total expenditure
p, I – are given

The result of model application: optimal quantities of consumption (x_{opt})

Discrete Choice Theory

Assumptions:

1. x **is discrete**

2. On 1 shopping trip a consumer chooses **1 unit of 1 type of good -> $\boldsymbol{x = 1}$** (very restrictive)

Example

3 cars: 1,2,3

Compare U_1, U_2, U_3.

If $U_1 > U_2 > U_3$, than car 1 is chosen.

The result of model application: probability of choice for each alternative i. ($\Pr(i)$)

From Discrete Choice and Consumer Theory to Multiple Discreteness Theory

Which theory to use when **on 1 shopping trip** consumer chooses:

1. Which goods to buy
   (discrete part of choice)

2. Which amount of each good to buy
   (continuous part of choice)

Approaches to modeling MD

Approach	Disadvantage
Enumerate all possible bundles of elemental choice alternatives Treat each bundle as a "composite" alternative within a traditional single discrete choice framework	Number of composite alternatives explodes as the number of elemental alternatives increases.
Use a multivariate statistical system, with several univariate DCM equations linked to each other through statistical correlations	"mechanical" statistical stitching of multiple univariate model equations rather than a unified, underlying theoretical work
Addresses both MD and quantity choice made at a single point in time (shopping trip). Addresses multiple consumption occasions over a succeeding time period. Tastes for products as well as number of consumption occasions can vary during time period -> an optimal purchase decision will include multiple alternatives in varing quatities (Hendel 1999, Dube 2004)	artificial method, not 100% routed in theory

Approaches to modeling MD

Approach Idea	**Disadvantages of approach**
More general approach. Uses classical microeconomic theory of constrained utility maximization. Two theoretically equivalent yet distinct approaches here: 1. The direct utility maximization approach (or Kuhn-Tucker approach) 2. The indirect utility approach	Not for all situation can be used. As not for all situations discrete and continuous parts are tightly coupled.

Approaches To Modeling Multiple Discreteness

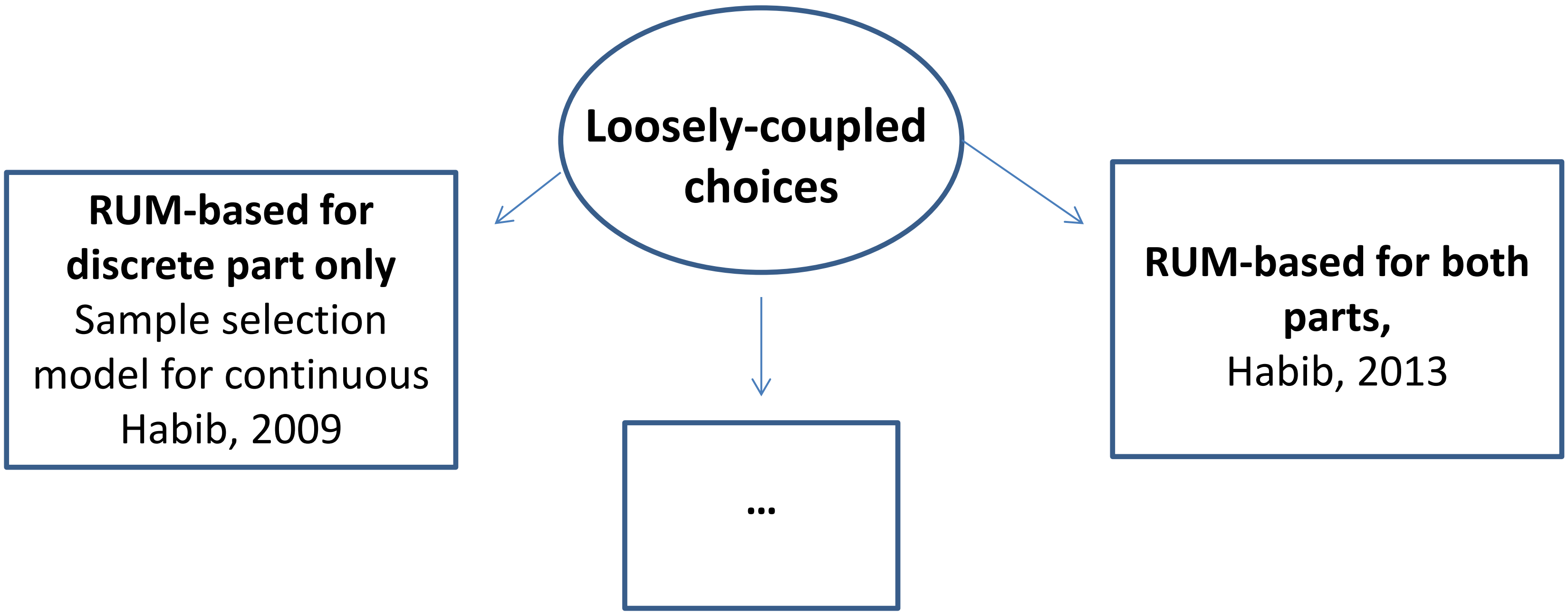
<table>
<tr><th>Structural</th><th>Non-structural (or reduced form)</th></tr>
<tr><td>Tightly coupled choices (discrete and continuous):
• 1 function is used to model each type of choice
• the connection between types of choices is implicitly addressed
• No extra measure is needed to address the correlation of parts

Completely random utility maximization (RUM)-based.</td><td>Loosely coupled choices:

• separate functions for discrete and continuous parts
• parts are endogenously related

• the correlation between 2 parts is modeled explicitly by capturing correlation between random elements</td></tr>
</table>

Types of Loosely Coupled Models



Literature: Heckman (1979), Dubin and McFadden (1984), Lee (1983), Munizaga, et.al. (2008), Habib (2009), Bhat (1998a, 2001), Ye and Pendyala(2009), Manchanda (1999), Baltas (2004), Edwards and Allenby (2003), Bhat and Shrinvasan (2005)

Structural Approach To Model MD: Ways to Derive Demand Functions

$$\begin{cases} U(p,x) \to max, \\ px = I, \\ x \geq 0 \end{cases}$$

$U(p,x)$ - random variable

1. **Kuhn-Tucker approach**

   $U(x)$ is random -> stochastic Kuhn-Tucker (KT) F.O.C. -> probabilities for consumption patterns

2. **Indirect utility approach**

   Specification of conditional indirect utility function.
   -> Demand functions are obtained via Roy's identity

Structural Approach: Directions For Research in MDC

$$\begin{cases} U(p,x) \to max, \\ px = I, \\ x \geq 0 \end{cases}$$

1. More flexible function forms for the utility specification

2. More flexible stochastic specifications for the utility functions

3. Greater flexibility in the specification of constraints faced by consumer

Structural Approach

Hanemann (1984)
1st work on MDC,
SDC case, linear utility function

Kim (2002)
Translated CES utility function
MDC + **Normal distribution for errors**

All are frightened of multivariate integrals!!!

Bhat (2005, 2008)
Translated CES utility function
Box-Cox transformation of utility function
EV Type I distribution for errors (MDCEV Model)

Lot's of applications appeared!!!

Generalizations of MDCEV:

1. **MDCNEV**
   Bhat, Pinjari (2010)
2. **MDCGEV**
   Pinjari (2011)
3. **MDC Cross-Nested EV**
   Pinjari (2011)
4. **Mixed MDCEV**
   Munger (2012)
5. To be continued…

Chandra R. Bhat’s Methodological Works

1. “A multiple discrete-continuous extreme value model: formulation and application to discretionary time-use decisions”, Bhat (2005)
2. “The multiple discrete-continuous extreme value (MDCEV) model: Role of utility function parameters, identification considerations, and model extensions”, Bhat (2008)

Aims of The Papers

1. To formulate a utility theory-based model for multiple discrete-continuous choice that assumes diminishing MU (satiation)
2. To clarify the role of all the parameters
3. To make empirics based on the Travel Survey data.

RUM Model

$$\begin{cases} U(x) \to max, \\ \sum_{k=1}^{K} e_k = E, e_k = p_k x_k, \\ x \geq 0 \end{cases}$$

$U(x)$ - direct utility function
$p = (p_1, \dots, p_N)$ – unit prices for goods $\mathrm{k} = \overline{1, K}$
$x = (x_1, \dots,\ x_N)^T$ – volumes of consumption
$e_k\ -$ total expenditures on good k
E –total expenditure

p, E – are given

Model: Utility Function

We need such utility function that:

1. is quasi-concave, increasing and continuously differentiable with respect to x (to get a solution)
2. is non-linear (to allow MD)
3. allows to model diminishing marginal rate (as more than 1 unit is bought)

Evolution of Utility Models

$$U = \sum_{j=1}^{K} \psi(x_j)(t_j + \gamma_j)^{\alpha_j}$$

(2005)

$$\boxed{U(x) = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k} \psi_k \left\{ \left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right\},}$$

(2008)

$\psi_k, \gamma_k, \alpha_k - parameters$
$\mathrm{k}\ - alternative, i = \overline{1, K}$
$x_k - volume\ of\ consumption\ of\ good\ k$

Role Of Utility Parameters: ψ_k

$$U(x) = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k} \psi_k \left\{ \left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right\},$$

$$\frac{\partial U(x)}{\partial x_k} = \psi_k \left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k - 1}.$$

1. Baseline marginal utility (BMU) at the point of zero consumption level.
2. A higher BMU for one of the goods will increase the overall utility from consumption of this good -> will imply less likelihood of corner solution for this good.

Role Of Utility Parameters: γ_k

$$U = \sum_{j=1}^{K} \psi(x_j)(t_j + \gamma_j)^{\alpha_j}$$

$$U(x) = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k} \psi_k \left\{ \left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right\},$$

1. Allows indifference curves to cross consumption axes (corner solutions)
2. Satiation parameter (indifference curves get steeper with the growth of parameter: consumer is willing to give more amount of 1 good to get 2 good)
3. The higher the value of γ_k, the more utility consumer gets from bigger amount of alternatives -> the less is the satiation

Role Of Utility Parameters: α_k

1. To reduce MU with increasing consumption of good k -> satiation parameter
2. The closer it to 1 the less is satiation. The closer to zero, the more satiation.

Empirical Identification Issues Associated With Utility Form (1)

$$U(x) = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k} \psi_k \left\{ \left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right\},$$

γ_k, α_k - both contribute satiation

Empirically it is difficult to disentangle these effects separately. ->

1. α_k - profile ($\gamma_k = 1$)
2. γ_k - profile ($\alpha_k = 0$)

Estimate both profiles and choose 1 that provides better fit.

Empirical Identification Issues Associated With Utility Form (2)

1. $\alpha_k \in [0,1] \rightarrow \alpha_k = [1 - \exp(-\delta_k)]$, $\delta_k = \theta_s^T y_s$

   y_s - vector of individual characteristics
   θ_s^T-vector of parameters

2. $\gamma_k > 0 \rightarrow \gamma_k = \exp(\mu_k)$, $\mu_k = \varphi_k^T w_k$

   w_k - vector of individual characteristics
   φ_s^T-vector of parameters

3. $\psi_k > 0 \rightarrow \psi_k = \exp\,(\beta^T z_k)$

   z_k -vector of attributes, characterizing alternative k
   β^T - vector of parameters

Kuhn-Tucker Approach

$$\mathscr{L} = \sum_k \frac{\gamma_k}{\alpha_k} \left[\exp(\beta' z_k + \varepsilon_k)\right] \left\{ \left(\frac{e_k}{\gamma_k p_k} + 1 \right)^{\alpha_k} - 1 \right\} - \lambda \left[\sum_{k=1}^{K} e_k - E \right],$$

KT F.O.C.:

$$\left[\frac{\exp\left(\beta' z_k + \varepsilon_k\right)}{p_k}\right] \left(\frac{e_k^*}{\gamma_k p_k} + 1 \right)^{\alpha_k - 1} - \lambda = 0, \quad \text{if } e_k^* > 0, \;\; k = 1, 2, \ldots, K,$$
$$\left[\frac{\exp\left(\beta' z_k + \varepsilon_k\right)}{p_k}\right] \left(\frac{e_k^*}{\gamma_k p_k} + 1 \right)^{\alpha_k - 1} - \lambda < 0, \quad \text{if } e_k^* = 0, \;\; k = 1, 2, \ldots, K.$$

$\varepsilon_k - i.i.d.\,EV\ Type\ 1$

Optimal Expenditure Allocation

$$V_k + \varepsilon_k = V_1 + \varepsilon_1 \quad \text{if } e_k^* > 0 \ \ (k = 2, 3, \ldots, K),$$
$$V_k + \varepsilon_k < V_1 + \varepsilon_1 \quad \text{if } e_k^* = 0 (k = 2, 3, \ldots, K), \ \text{where}$$
$$V_k = \beta' z_k + (\alpha_k - 1) \ln \left(\frac{e_k^*}{\gamma_k p_k} + 1 \right) - \ln p_k \quad (k = 1, 2, 3, \ldots, K).$$

The probability that individual will allocate expenditure in M of the K goods is:

$$P(e_1^*, e_2^*, e_3^*, \ldots, e_M^*, 0, 0, \ldots, 0) = \frac{1}{\sigma^{M-1}} \left[\prod_{i=1}^{M} c_i \right] \left[\sum_{i=1}^{M} \frac{1}{c_i} \right] \left[\frac{\prod_{i=1}^{M} e^{V_i/\sigma}}{\left(\sum_{k=1}^{K} e^{V_k/\sigma} \right)^M} \right] (M-1)!$$

The probability of consumption pattern of the goods:

$$P(x_1^*, x_2^*, x_3^*, \ldots, x_M^*, 0, 0, \ldots, 0) = \frac{1}{p_1} \cdot \frac{1}{\sigma^{M-1}} \left[\prod_{i=1}^{M} f_i \right] \left[\sum_{i=1}^{M} \frac{p_i}{f_i} \right] \left[\frac{\prod_{i=1}^{M} e^{V_i/\sigma}}{\left(\sum_{k=1}^{K} e^{V_k/\sigma} \right)^M} \right] (M-1)!$$

Model With An Outside Good

The utility of the model is not attribute specific, but alternative specific. ->

The optimization problems becomes very difficult with the growth of number of parameters. ->

Let’s include in the model only relevant alternatives and other alternative combine in the outside good.

Essiantial Hicksian composite good

$$U(x) = \frac{1}{\alpha_1}\exp(\varepsilon_1)x_1^{\alpha_1} + \sum_{k=2}^{K}\frac{\gamma_k}{\alpha_k}\exp(\beta' z_k + \varepsilon_k)\left\{\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k} - 1\right\}.$$

Empirics: Data

1. 2 000 San Francisco Bay Area Travel Survey
2. 3 500 h/h with non-zero vehicle ownership
3. Vehicles are grouped in 5 categories:
   – Passenger car
   – Sports Utility Vehicle (SUV)
   – Pickup
   – Minivan
   – Van
4. Vehicle miles for each type of transport/

Thank you for attention!

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